Chapter Eleven

Algebraic Ratio and Proportion

It is important for us to have a clear concept of ratio and proportion. Arithmetical ratio and proportion have been elaborately discussed in class VII. In this chapter, algebraic ratio and proportion will be discussed. We regularly use the concept of ratio and proportion in construction materials and in the production of food staff, in consumers production, in using fertilizer in land, in making the shapes and design of many things attractive and good tooking and in many areas of our daily activities. Many problems of daily lives can be solved by using ratio and proportion.

At the end of this chapter, the students will be able to :

- Explain algebraic ratio and proportion.
- Use different types of rules of transformation related to proportion.
- Describe successive proportion.
- Use ratio, proportion, successive proportion in solving real lives problem.

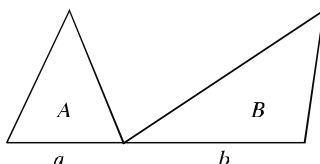
11·1 Ratio

Of two quantities of same kind and unit, how many times or parts of other can be expressed by a fraction. This fraction is called the ratio of two quantities.

The ratio of two quantities p and q is written in $p : q = \frac{p}{q}$. The quantities p and q are to be of same kind and same unit. p is called antecedent and q is called subsequent of the ratio.

Some times we use ratio in approximate measure. Such as, the number of cars on the road at 8 A.M. doubles the number at 10 A.M. In this case, it is not necessary to know the exact number of cars to determine the ratio. Again, in many occasions, we say that the area of your house is three times the area of mine. Here, also it is not necessary to know the exact area of the house. We use the concept of ratio in cases of practical life.

11·2 Proportion

If four quantities are such that the ratio of first and second quantities is equal to the ratio of third and fourth quantities, those four quantities form a proportion. If a, b, c, d are four such quantities, we write $a : b = c : d$. The four quantities of proportion need not to be of same kinds. The two quantities of the ratio are to be of the same kind only.

In the above figure, let the base of two triangles be a and b respectively and their height is h unit. If the areas of the triangle be A and B square units, we can write,

$$\frac{A}{B} = \frac{\frac{1}{2}ah}{\frac{1}{2}bh} = \frac{a}{b} \quad \text{or } A : B = a : b$$

i.e. ratio of two areas is equal to ratio of two bases.

Ordered proportional

By ordered proportional of a, b, c it is meant that $a : b = b : c$.

a, b, c will be ordered proportional if and only if $b^2 = ac$. In case of ordered proportional, all the quantities are to be of same kinds. In this case, c is called third proportional of a and b and b is called mid-proportional of a and c.

Example 1. A and B traverses fixed distance in t_1 and t_2 minutes. Find the ratio of average velocity of A and B.

Solution : Let the average velocities of A and B be v_1 sec/metre and v_2 sec/metre respectively. So, in time t_1 minutes A traverses v_1t_1 metres and in t_2 minutes B traverses the distance v_2t_2 meters.

According to the problem, $v_1t_1 = v_2t_2$ $\therefore \frac{v_1}{v_2} = \frac{t_2}{t_1}$

Here, ratio of the velocities is inversely proportional to the ratio of time.

Activity: 1. Express 3.5 : 5.6 into 1 : a and b : 1 2. If $x : y = 5 : 6$, $3x : 5y =$ What ?

11·3 Transformation of Ratio

Here, the quantities of ratios are positive numbers.

(1) If $a : b = c : d$ then $b : a = d : c$ [Invertendo]

Proof : Given that, $\frac{a}{b} = \frac{c}{d}$

$\therefore ad = bc$ [multiplying both the sides by bd]

or, $\frac{ad}{ac} = \frac{bc}{ac}$ [dividing both the sides by ac where $a \neq 0, c \neq 0$]

or, $\frac{d}{c} = \frac{b}{a}$

i.e., $b : a = d : c$

(2) If $a : b = c : d$ then $a : c = b : d$ [alternendo]

Proof : Given that, $\frac{a}{b} = \frac{c}{d}$

$\therefore ad = bc$ [multiplying both the sides by bd]

or, $\frac{ad}{cd} = \frac{bc}{cd}$ [dividing both the sides by cd where $c \neq 0, d \neq 0$]

or, $\frac{a}{c} = \frac{b}{d}$

i.e., $a : c = b : d$

(3) If $a : b = c : d$ then $\frac{a+b}{b} = \frac{c+d}{d}$ [componendo]

Proof : Given that, $\frac{a}{b} = \frac{c}{d}$

$\therefore \frac{a}{b} + 1 = \frac{c}{d} + 1$ [Adding 1 to both the sides]

i.e., $\frac{a+b}{b} = \frac{c+d}{d}$

(4) If $a : b = c : d$ then $\frac{a-b}{b} = \frac{c-d}{d}$ [dividendo]

Proof : $a : b = c : d$

$\therefore \frac{a}{b} - 1 = \frac{c}{d} - 1$ [subtracting 1 from both the sides]

i.e., $\frac{a-b}{b} = \frac{c-d}{d}$

(5) If $a : b = c : d$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ [componendo -dividendo]

Proof : Given that, $\frac{a}{b} = \frac{c}{d}$

By componendo, $\frac{a+b}{b} = \frac{c+d}{d}$..............($i$)

Again by dividendo, $\frac{a-b}{b} = \frac{c-d}{d}$

or, $\frac{b}{a-b} = \frac{d}{c-d}$ [by invertendo]............. (ii)

Therefore, $\frac{a+b}{b}\times\frac{b}{a-b}=\frac{c+d}{d}\times\frac{d}{c-d}$ [multiplying (i) and (ii)]

i.e., $\frac{a+b}{a-b}=\frac{c+d}{c-d}$. [here $a\neq b$ and $c\neq d$]

(6) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$ then each of the ratio $=\frac{a+c+e+g}{b+d+f+h}$.

Proof : Let, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=k.$

$\therefore a=bk,\quad c=dk,\quad e=fk,\quad g=hk$

$\therefore \frac{a+c+e+g}{b+d+f+h}=\frac{bk+dk+fk+hk}{b+d+f+h}=\frac{k(b+d+f+h)}{b+d+f+h}=k,$

But k is equal to each of the ratio.

$\therefore \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{a+c+e+g}{b+d+f+h}.$

Activity : 1. Sum of ages of mother and sister is s years. Before t years, the ratio of their ages was $r:p$. What will be the ratio of their ages after x years ?
2. The shadow of a man of height r metre, standing at p metre from a lightpost is s metre. If the height of the lightpost be h metre, what was the distance of the man from the lightpost ?

Example 2. The ratio of present ages of father and son is $7:2$, and the ratio will be $8:3$ after 5 years. What are their present ages ?

Solution : Let the present age of father be a and that of son is b. So, by the conditions of first and second of the problems, we have,

$\frac{a}{b}=\frac{7}{2}$...........(i)

$\frac{a+5}{b+5}=\frac{8}{3}$...........$(ii)$

From equation (i), we get,

$a=\frac{7b}{2}$...........(iii)

From equation (ii), we get,

$3(a+5)=8(b+5)$

or, $3a+15=8b+40$

or, $3a-8b=25$

or, $3\times\frac{7b}{2}-8b=25$ [by using (iii)]

or, $\dfrac{21b-16b}{2}=25$

or, $5b=50$

$\therefore\ b=10$

Putting $b=10$ in equation (*iii*), we get, $a=35$

$\therefore$ The present age of father is 35 years, and that of son is 10 years.

Example 3. If $a:b=b:c$, prove that $\left(\dfrac{a+b}{b+c}\right)^2=\dfrac{a^2+b^2}{b^2+c^2}$.

Solution : Given that, $a:b=b:c$

$\therefore\ b^2=ac$

Now, $\left(\dfrac{a+b}{b+c}\right)^2=\dfrac{(a+b)^2}{(b+c)^2}$

$$=\frac{a^2+2ab+b^2}{b^2+2bc+c^2}$$

$$=\frac{a^2+2ab+ac}{ac+2bc+c^2}$$

$$=\frac{a(a+2b+c)}{c(a+2b+c)}=\frac{a}{c}$$

and $\dfrac{a^2+b^2}{b^2+c^2}=\dfrac{a^2+ac}{ac+c^2}$

$$=\frac{a(a+c)}{c(a+c)}$$

$$=\frac{a}{c}$$

$\therefore\ \left(\dfrac{a+b}{b+c}\right)^2=\dfrac{a^2+b^2}{b^2+c^2}$

Example 4. If $\dfrac{a}{b}=\dfrac{c}{d}$, show that $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{ac+bd}{ac-bd}$.

Solution : Let, $\dfrac{a}{b}=\dfrac{c}{d}=k$; $\therefore\ a=bk$ and $c=dk$

Now, $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{(bk)^2+b^2}{(bk)^2-b^2}=\dfrac{b^2(k^2+1)}{b^2(k^2-1)}=\dfrac{k^2+1}{k^2-1}$

and $\dfrac{ac+bd}{ac-bd}=\dfrac{bk\cdot dk+bd}{bk\cdot dk-bd}=\dfrac{bd(k^2+1)}{bd(k^2-1)}=\dfrac{k^2+1}{k^2-1}$

$\therefore\ \dfrac{a^2+b^2}{a^2-b^2}=\dfrac{ac+bd}{ac-bd}$.

Example 5. Solve : $\frac{1-ax}{1+ax}\sqrt{\frac{1+bx}{1-bx}} = 1, \quad 0 < b < 2a < 2b.$

Solution : Given that, $\frac{1-ax}{1+ax}\sqrt{\frac{1+bx}{1-bx}} = 1$

$\therefore \sqrt{\frac{1+bx}{1-bx}} = \frac{1+ax}{1-ax}$

or, $\frac{1+bx}{1-bx} = \frac{(1+ax)^2}{(1-ax)^2}$ [squaring both the sides]

or, $\frac{1+bx}{1-bx} = \frac{1+2ax+a^2x^2}{1-2ax+a^2x^2}$

or, $\frac{1+bx+1-bx}{1+bx-1+bx} = \frac{1+2ax+a^2x^2+1-2ax+a^2x^2}{1+2ax+a^2x^2-1+2ax-a^2x^2}$ [by componendo and dividendo]

or, $\frac{2}{2bx} = \frac{2(1+a^2x^2)}{4ax}$

or, $\frac{1}{bx} = \frac{1+a^2x^2}{2ax}$

or, $2ax = bx(1+a^2x^2)$

or, $x\{2a - b(1+a^2x^2)\} = 0$

$\therefore$ Either $x = 0$ or $2a - b(a+a^2x^2) = 0$

or, $b(1+a^2x^2) = 2a$

or, $1+a^2x^2 = \frac{2a}{b}$

or, $a^2x^2 = \frac{2a}{b} - 1$

or, $x^2 = \frac{1}{a^2}\left(\frac{2a}{b} - 1\right)$

$\therefore x = \pm\frac{1}{a}\sqrt{\frac{2a}{b} - 1}$

$\therefore$ Required solution $x = 0$, $x = \pm\frac{1}{a}\sqrt{\frac{2a}{b} - 1}$.

Example 6. If $\frac{6}{x} = \frac{1}{a} + \frac{1}{b}$, show that $\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = 2, \; a \neq b.$

Solution : Given that, $\frac{6}{x}=\frac{1}{a}+\frac{1}{b}$

$\therefore 6ab=(a+b)x$ [multiplying both the sides by abx]

i.e., $x=\frac{6ab}{(a+b)}$

or, $\frac{x}{3a}=\frac{2b}{a+b}$

$\therefore \frac{x+3a}{x-3a}=\frac{2b+a+b}{2b-a-b}$ [by componendo and dividendo]

or, $\frac{x+3a}{x-3a}=\frac{a+3b}{b-a}$

Again, $\frac{x}{3b}=\frac{2a}{a+b}$

or, $\frac{x+3b}{x-3b}=\frac{2a+a+b}{2a-a-b}$ [by componendo and dividendo]

$\therefore \frac{x+3b}{x-3b}=\frac{3a+b}{a-b}$

Now, $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$

$=\frac{a+3b}{b-a}-\frac{3a+b}{b-a}=\frac{a+3b-3a-b}{b-a}=\frac{2(b-a)}{b-a}=2$.

$\therefore \frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=2.$

Example 7. If $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=p$, prove that, $p^2-\frac{2p}{x}+1=0.$

Solution : Given that, $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=p$

$\therefore \frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}}=\frac{p+1}{p-1}$ [by componendo -dividendo]

or, $\frac{2\sqrt{1+x}}{2\sqrt{1-x}}=\frac{p+1}{p-1}$ or, $\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{p+1}{p-1}$

or, $\frac{1+x}{1-x}=\frac{(p+1)^2}{(p-1)^2}=\frac{p^2+2p+1}{p^2-2p+1}$ [squaring both sides]

or, $\frac{1+x+1-x}{1+x-1+x}=\frac{p^2+2p+1+p^2-2p+1}{p^2+2p+1-p^2+2p-1}$ [by componendo -dividendo]

or, $\frac{1}{x}=\frac{p^2+1}{2p}$ or, $p^2+1=\frac{2p}{x}$

$\therefore$ $p^2-\frac{2p}{x}+1=0.$

Example 8. If $\frac{a^3+b^3}{a-b+c}=a(a+b)$, prove that a, b, c are ordered proportional.

Solution : Given that, $\frac{a^3+b^3}{a-b+c}=a(a+b)$

or, $\frac{a^3+b^3}{a-b+c}=a(a+b)$

or, $\frac{(a+b)(a^2-ab+b^2)}{a-b+c}=a(a+b)$

or, $\frac{a^2-ab+b^2}{a-b+c}=a$ [Dividing both sides by ($a+b$)]

or, $a^2-ab+b^2=a^2-ab+ac$

$\therefore$ $b^2=ac$

$\therefore$ a, b, c are ordered proportional.

Example 9. If $\frac{a+b}{b+c}=\frac{c+d}{d+a}$, prove that $c=a$ or $a+b+c+d=0$.

Solution : Given that, $\frac{a+b}{b+c}=\frac{c+d}{d+a}$

or, $\frac{a+b}{b+c}-1=\frac{c+d}{d+a}-1$ [subtracting 1 from both the sides]

or, $\frac{a+b-b-c}{b+c}=\frac{c+d-d-a}{d+a}$

or, $\frac{a-c}{b+c}=\frac{c-a}{d+a}$

or, $\frac{a-c}{b+c}+\frac{a-c}{d+a}=0$

or, $(a-c)\left(\frac{1}{b+c}+\frac{1}{d+a}\right)=0$

or, $(a-c)\dfrac{(d+a+b+c)}{(b+c)(d+a)}=0$

or, $(a-c)(d+a+b+c)=0$

$\therefore$ Either $a-c=0$ i.e., $a=c$

or, $a+b+c+d=0$.

Example 10. If $\dfrac{x}{y+z}=\dfrac{y}{z+x}=\dfrac{z}{x+y}$ and x, y, z are not mutually equal, prove that the value of each ratio is either equal -1 or equal $\dfrac{1}{2}$.

Solution : Let, $\dfrac{x}{y+z}=\dfrac{y}{z+x}=\dfrac{z}{x+y}=k$

$\therefore x=k(y+z)$................(i)

$y=k(z+x)$...................$(ii$

$z=k(x+y)$...................(iii)

Subtracting (ii) from (i), we get,

$x-y=k(y-x)$ or, $k(y-x)=-(y-x)$

$\therefore\ k=-1$

Again, adding (i), (ii) and (iii), we get,

$x+y+z=k(y+z+z+x+x+y)=2k(x+y+z)$

$\therefore\ \ \text{k}=\dfrac{1(x+y+z)}{2(x+y+z)}=\dfrac{1}{2}$

$\therefore$ Therefore, the value of each of the ratio is -1 or $\dfrac{1}{2}$.

Example 11. If $ax=by=cz$, show that $\dfrac{x^2}{yz}+\dfrac{y^2}{zx}+\dfrac{z^2}{xy}=\dfrac{bc}{a^2}+\dfrac{ca}{b^2}+\dfrac{ab}{c^2}$.

Solution : Let, $ax=by=cz=k$

$\therefore\ x=\dfrac{k}{a},\ \ y=\dfrac{k}{b},\ \ z=\dfrac{k}{c}$

Now, $\dfrac{x^2}{yz}+\dfrac{y^2}{zx}+\dfrac{z^2}{xy}=\dfrac{k^2}{a^2}\times\dfrac{bc}{k^2}+\dfrac{k^2}{b^2}\times\dfrac{ca}{k^2}+\dfrac{k^2}{c^2}\times\dfrac{ab}{k^2}=\dfrac{bc}{a^2}+\dfrac{ca}{b^2}+\dfrac{ab}{c^2}$

i.e., $\dfrac{x^2}{yz}+\dfrac{y^2}{zx}+\dfrac{z^2}{xy}=\dfrac{bc}{a^2}+\dfrac{ca}{b^2}+\dfrac{ab}{c^2}$.

Exercise 11·1

1. If the sides of two squares be a and b metres respectively, what will be the ratio of their areas ?
2. If the area of a circle is equal to the area of a square, find the ratio of their paremetre.
3. If the ratio of two numbers is 3 : 4 and their L.C.M. is 180, find the two numbers.
4. The ratio of absent and present students of a day in your class is found to be 1:4. Express the number in percentage of absent students in terms of total students.
5. A thing is bought and sold at the loss of 28%. Find the ratio of buying and selling cost.
6. Sum of the ages of father and son is 70 years. 7 years ago, the ratio of their ages were 5 : 2. What will the ratio of their ages be after 5 years.
7. If $a : b = b : c$, prove that,

   (i) $\frac{a}{c} = \frac{a^2+b^2}{b^2+c^2}$ $\qquad (ii)$ $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right) = a^3+b^3+c^3$

   (iii) $\frac{abc(a+b+c)^3}{(ab+bc+ca)^3} = 1$ $\qquad (iv)$ $a-2b+c = \frac{(a-b)^2}{a} = \frac{(b-c)^2}{c}$

8. Solve : (i) $\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}} = \frac{1}{3}$ $\qquad (ii)$ $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = b$

   (iii) $\frac{a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}} = \frac{b}{x}$, $2a > b > 0$ and $x \neq 0$.

   (iv) $\frac{\sqrt{x-1}+\sqrt{x-6}}{\sqrt{x-1}-\sqrt{x-6}} = 5$ $\qquad (v)$ $\frac{\sqrt{ax+b}+\sqrt{ax-b}}{\sqrt{ax+b}-\sqrt{ax-b}} = c$

   (vi) $81\left(\frac{1-x}{1+x}\right)^3 = \frac{1+x}{1-x}$

9. If $\frac{a}{b} = \frac{c}{d}$, show that, (i) $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{c^2+cd+d^2}{c^2-cd+d^2}$

   (ii) $\frac{ac+bd}{ac-bd} = \frac{c^2+d^2}{c^2-d^2}$

10. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, show that,

(i) $\dfrac{a^3+b^3}{b^3+c^3}=\dfrac{b^3+c^3}{c^3+d^3}$ (ii) $(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$

11. If $x=\dfrac{4ab}{a+b}$, show that, $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}=2, \quad a\neq b.$

12. If $x=\dfrac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$, prove that, $x^3-3mx^2+3x-m=0$

13. If $x=\dfrac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$, show that, $3bx^2-4ax+3b=0.$

14. If $\dfrac{a^2+b^2}{b^2+c^2}=\dfrac{(a+b)^2}{(a+c)^2}$, prove that, a, b, c are ordered proportional.

15. If $\dfrac{x}{b+c}=\dfrac{y}{c+a}=\dfrac{z}{a+b}$, prove that, $\dfrac{a}{y+z-x}=\dfrac{b}{z+x-y}=\dfrac{c}{x+y-z}.$

16. If $\dfrac{bz-cy}{a}=\dfrac{cx-az}{b}=\dfrac{ay-bx}{c}$, prove that, $\dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}.$

17. If $\dfrac{a+b-c}{a+b}=\dfrac{b+c-a}{b+c}=\dfrac{c+a-b}{c+a}$ and $a+b+c\neq 0$, prove that, $a=b=c.$

18. If $\dfrac{x}{xa+yb+zc}=\dfrac{y}{ya+zb+xc}=\dfrac{z}{za+xb+yc}$ and $x+y+z\neq 0$, show that, each of the ratio is $=\dfrac{1}{a+b+c}.$

19. If $(a+b+c)p=(b+c-a)q=(c+a-b)r=(a+b-c)s$, prove that, $\dfrac{1}{q}+\dfrac{1}{r}+\dfrac{1}{s}=\dfrac{1}{p}.$

20. If $lx=my=nz$, show that, $\dfrac{x^2}{yz}+\dfrac{y^2}{zx}+\dfrac{z^2}{xy}=\dfrac{mn}{l^2}+\dfrac{nl}{m^2}+\dfrac{lm}{n^2}.$

21. If $\dfrac{p}{q}=\dfrac{a^2}{b^2}$ and $\dfrac{a}{b}=\dfrac{\sqrt{a+q}}{\sqrt{a-q}}$, show that, $\dfrac{p+q}{a}=\dfrac{p-q}{q}.$

11·4. Successive Ratio

Let Rani's earning be Tk. 1000, Soni's earning be Tk. 1500 and Somi's earning be Tk. 2500. Here, Rani's earning : Soni's earning = 1000 : 1500 = 2 : 3 ; Soni's

earning : Sami's earning = 1500 : 2500 = 3 : 5. Hence, Runi's : Soni's : Sami's earning = 2 : 3 : 5.

If two ratios are of the form *a : b* and *b : c*, they can be put in the form *a : b : c*. This is called successive ratio. Any two or more than two ratios can be put in this form. It is to be noted that if two ratios are to be put in the form *a : b c*, antecedent of the first ratio and subsequent of the second ratio are to be equal. Such as, if two ratios 2 : 3 and 4 : 3 are to be put in the form *a : b* : *c* the subsequent quantity of first ratio is to be made equal to antecedent quantity of the second ratio. That is those quantities are to be made equal to their L.C.M.

Here, $2:3=\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}=8:12$ Again, $4:3=\frac{4}{3}=\frac{4\times3}{3\times3}=\frac{12}{9}=12:9$

Therefore, if the ratios 2 : 3 and 4 : 3 are put in the form, *a : b : c* will be 8 : 12 : 9.

It is to be noted that if the earning of Sami in the above example is 1125, the ratio of their earnings will be 8 : 12 : 9.

Example 12. If *a, b, c* are quantities of same kind and *a : b* = 3 : 4, *b : c* = 6 : 7, what will be *a : b : c* ?

Solution : $\frac{a}{b}=\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$ and $\frac{b}{c}=\frac{6}{7}=\frac{6\times2}{7\times2}=\frac{12}{14}$ [L.C.M. of 4 and 6 is 12]

$\therefore a : b : c = 9 : 12 : 14$.

Example 13. The ratio of angles of a triangle is 3 : 4 : 5. Express the angles in degree.

Solution : Sum of three angles $= 180^\circ$

Let the angles, according to given ratio, be $3x$, $4x$ and $5x$.

According to the problem, $3x + 4x + 5x = 180^\circ$ or, $12x = 180^\circ$ or, $x = 15^\circ$

Therefore, the angles are $3x = 3\times15^\circ = 45^\circ$

$4x = 4\times15^\circ = 60^\circ$

and $5x = 5\times15^\circ = 75^\circ$

Example 14. If the sides of a square increase by 10%, how much will the area be increased in percentage ?

Solution : Let each side of the square be a metre

$\therefore$ Area of the square be a^2 square metre

If the side increases by 10%, each side will be $(a + 10\% \text{ of } a)$ metre or $1{\cdot}10a$ metre.

In this case, the area of the square will be $(1\cdot10a)^2$ square metre or, $1\cdot21a^2$ square metre.

Area increases by $(1\cdot21a^2 - a^2) = 0.21a^2$ square metre

$\therefore$ The percentage of increment of the area will be $\frac{0\cdot21a^2}{a^2} \times 100\% = 21\%$

Activity :

1. There are 35 male and 25 female students in your class. The ratios of rice and pulse are 3 : 1 and 5 : 2 given by each of the male and female students for taking khisuri in a picnic. Find the ratio of total rice and total pulse.

11·5 Proportional Division

Division of a quantity into fixed ratio is called proportional division. If S is to be divided into $a : b : c : d$, dividing S by $(a + b + c + d)$ the parts a, b, c and d are to be taken.

Therefore,

1st part $= \frac{a}{a+b+c+d}$ of $S = \frac{Sa}{a+b+c+d}$

2nd part $= \frac{b}{a+b+c+d}$ of $S = \frac{Sb}{a+b+c+d}$

3rd part $= \frac{c}{a+b+c+d}$ of $S = \frac{Sc}{a+b+c+d}$

4th part $= \frac{d}{a+b+c+d}$ of $S = \frac{Sd}{a+b+c+d}$

In this way, any quantity may be divided into any fixed ratio.

Example 15. Divide Tk. 5100 among 3 persons in such a way that 1st person's part : 2nd person's part : 3rd person's part are $= \frac{1}{2} : \frac{1}{3} : \frac{1}{9}$.

Solution : Here, $\frac{1}{2} : \frac{1}{3} : \frac{1}{9} = \left(\frac{1}{2} \times 18\right) : \left(\frac{1}{3} \times 18\right) : \left(\frac{1}{9} \times 18\right)$ [L.C.M. of 2, 3, 9 is 18]

$= 9 : 6 : 2$

Sum of the quantities of ratio $= 9 + 6 + 2 = 17$.

1st person's part $= \text{Tk. } 5100 \times \frac{9}{17} = \text{Tk. } 2700$

2nd person's part $= \text{Tk. } 5100 \times \frac{6}{17} = \text{Tk. } 1800$

3rd person's part $= \text{Tk. } 5100 \times \frac{2}{17} = \text{Tk. } 600$

Therefore, three persons will have Tk. 2700, Tk. 1800 and Tk. 600 respectively.

Exercise 11·2

1. If a, b, c are ordered proportional, which one is correct of the followings ?

   (a) $a^2 = bc$ (b) $b^2 = ac$ (c) $ab = bc$ (d) $a = b = c$

2. The ratio of ages of Arif and Akib is 5 : 3 ;if Arif is of 20 years old, how many years later the ratio of their ages will be 7 : 5. ?

   (a) 5 years (b) 6 years (c) 8 years (d) 10 years

3. Note the following information :

   (i) All the four quantities need not to be of same kind in proportion.

   (ii) The ratio of areas of two triangles is equal to the ratio of areas of their bases.

   (iii) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$, value of each ratio will be $\frac{a+c+e+g}{b+d+f+h}$.

On the basis of above information, which one of the followings is correct?

(a) (i) and (ii) (b) (ii) and (iii)

(c) (i) and (iii) (d) (i), (ii) and (iii)

The ratio of angles of ΔABC is 2 : 3 : 5 and the ratio of angles of the quadrilateral $ABCD$ is 3 : 4 : 5 : 6. Based on this information, answer questions number 4 and 5.

4. If the sides of a square double, how much will the area of a square be increased?

(a) 2 times (b) 4 times

(c) 8 times (d) 6 times

5. If $x : y - 7 : 5,\ y : z = 5 : 7,\ z : x =$ how much ?

6. The estimated cost for the construction of a wooden bridge is Tk. 90,000. But Tk. 21,000 has been spent more. What is the percentage of the excess cost ?

7. The ratio of rice and husk in paddy to 7 : 3. What is the percentage of rice in it ?

8. The weight ot 1 cubic cm. wood is 7 decigram. What is the percentage of the weight of wood to the equivalent volume of water ?

9. Distribute Tk. 300 among *a, b, c, d* in such a way that the raios are *a*'s part : *b*'s part = 2 : 3, *b*'s part : *c*'s part = 1 : 2 and *c*'s part : *d*'s part 3 : 2.

10. Three fishermen have caught 690 pieces of fishes. The ratios of their parts are $\frac{2}{3}, \frac{4}{5}$ and $\frac{5}{6}$. How many fishes each of them will get. ?

11. The parametre of a triangle is 45 cm. The ratio of the lengths of the sides is 3 : 5 : 7. Find the length of each sides.

12. Distribute Tk. 1011 in the ratio $\frac{3}{4} : \frac{4}{5} : \frac{6}{7}$.

13. If the ratio of two numbers is 5 : 7 and their H.C.F. is 4, what is L.C.M. of the numbers.

14. In a cricket game, the total runs scored by Sakib, Mushfique and Mashrafi were 171. The ratio of runs scored by Sakib and Mushfiques, and Mushfique and Mashrafi was 3 : 2. What were the runs scored by them individually.
15. In a office, there were 2 officers, 7 clarks and 3 bearers. If a bearer gets Tk. 1, a clerk gets Tk. 2 and an officer gets Tk. 4. Their total salary is Tk. 15,000. What is their individual salary ?
16. In selecting the leader of a society, Mr. Denal won in ratio of 4 : 3 votes of the two contestants. If total numbers of members were 581 and 91 members did not cast their votes, what was the difference of votes by which opposite of Mr. Denal had been defeated ?
17. If the sides of a square are increased by 20%, what is percentage of increment of the area of the square ?
18. If the length of a rectangle is increased by 10% and the breadth is decreased by 10%, what is the percentage of increase or decrease of the area of the rectangle?
19. In a field, the ratio of production is 4 : 7 before and after irrigation. In that field, the production of paddy in a land previously was 304 quintal. What would be the production of paddy after irrigation ?
20. If the ratio of paddy and rice produced from paddy is 3 : 2 and the ratio of wheat and suzi produced from wheat is 4 : 3, find the ratio of rice and suzi produced from equal quantity of rice and wheat.
21. The are of a land is 432 square metre. If the ratios of lengths and breadths of that land and that of another land be 3 : 4 and 2 : 5 respectively, that what is the area of another land ?
22. Zami and Simi take loans of different amounts at the rate of 10% simple profit on the same day from same Bank. The amount on capital and profit which Zimi refunds after two years, the same amount Simi refunds after three years on capital and profit. Find the ratio of their loan.
23. The ratio of sides of a triangle is 5 : 12 : 13 and parametre is 30 cm.

    (a) Draw the triangle and write what type of triangle in respect of angles.

(b) Determine the area of a square drawn with the diagonal of a rectangle taking greater side as length and smaller side as breadth as the sides of a square.

(c) If the length is increased by 10% and breadth is increased by 20%, what will be percentage of increase of the area ?

24. The ratio of present and absent of students of a day in a class is 1 : 4.

(a) Express the percentage of absent students against total students.

(b) The ratio of present and absent students would be 1 : 9 if 10 more students were present. What was the total number of students ?

(c) Of the total number of students, the number of female students is less than male students by 20. Find the ratio of male and female students.